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FISCAL POLICY IN A STOCHASTIC MODEL OF ENDOGENOUS GROWTH

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ABSTRACT

It is nowadays widely believed that public schooling may contribute favourably to long-term economic growth. The income tax rates that are needed to finance government spending typically show an erratic time pattern. Such tax randomness could increase the intensity of the business cycle. Thus, government spending on education may spur economic growth, but the other side of the coin is that this is likely to increase the intensity of cyclical fluctuations. These issues are discussed in the context of a stochastic endogenous growth model with learning-by-doing as well as schooling activity. The key results are: (i) income taxation may go hand in hand with increased economic growth under certain conditions, (ii) tax randomness is responsible for a modest fraction of cyclical variability, (iii) the inclusion of stochastic taxation brings the model closer to the U.S. business cycle experience, (iv) the employment variability puzzle can be solved by introducing stochastic discounting, (v) the latter model can successfully pass a Wald-test, (vi) the interaction between long-term economic growth and the business cycle can be positive as well as negative, and (vii) the model typically suggests that capital taxes stabilize the economy.

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1. Introduction

This paper investigates the contribution of government policy to cyclical fluctuations and long-run economic growth. The idea is that the government provides services that facilitate productivity growth. These services include schooling, labour market programs, vocational training, but also the social and physical infrastructure, the quality of the environment, public health care, protection of property rights and safety. At least part of this public spending is financed by levying income taxation. It has been documented in a number of studies that observed effective marginal tax rates move erratically over time. Such tax randomness may in turn contribute to cyclical variability. To put it differently, government policy may spur economic growth, but the other side of the coin is that this is likely to increase the intensity of business cycle fluctuations. These issues are discussed in the context of a stochastic endogenous growth model with learning-by-doing as well as productive government spending. To fix ideas, attention is focused on public spending contributing to knowledge formation, such as formal and informal schooling, vocational training programs, but also applied research, and reorganization activity. Let us briefly refer to these activities as "schooling", or learning-or-doing.

Our main findings are the following. Long-term economic growth has an ambiguous relationship with the marginal capital tax rate and the marginal tax rate on labour income. Income taxation has repercussions on schooling, on labour supply, and on the allocation of employees to production and schooling activity. An increase in capital and labour taxes will intensify schooling activity; this will increase economic growth. The interaction between capital taxation and labour supply is ambiguous, however. It is shown that households may under some conditions increase their labour supply in response to higher capital taxation. What is at work here, is a negative wealth effect. Knowledge is a public good, and households do not take the knowledge-enhancing aspects of public schooling into account. Tax revenues used for educational spending are thus interpreted by the household as a loss (a negative wealth effect), which induces the household to reduce the demand for leisure. Labour taxation will typically induce households to shift to non-taxable leisure time, so that labour supply and knowledge accumulation from learning-by-doing will decline. Thirdly, for given labour supply, an increase in schooling activity will crowd out production activity. Overall, the effects from taxation on labour supply in combination with the crowding-out

effect of schooling will determine the household's occupation in leisure, production, and schooling activity. The analysis further suggests that tax randomness is responsible for a modest fraction of cyclical variability. It is shown that the inclusion of stochastic taxation brings the model closer to the U.S. business cycle experience: the artificial economy can explain simultaneously observed cyclical variability in production, consumption, savings, employment, and physical capital, as well as the observed cross-correlation of output with employment. These results suggest that the model constructed in this paper can compete successfully with existing real business cycle models. However, as in most real business cycle models, our artificial economy typically underestimates labour market fluctuations. We propose to solve the employment variability puzzle by introducing stochastic discounting. Indeed, the latter model can successfully pass a Wald-test, whereas models without it are always rejected. Two other interesting features of our model are that the interaction between long-term economic growth and the business cycle can be positive as well as negative, and that capital taxes stabilize the economy.

The analysis in this study is positive rather than normative. The marginal tax rates on capital and labour as well as the composition of government spending is taken as exogenously given. Consequently, for given labour supply, the division of market activity into production time and schooling is determined by the government's expenditures on education. In Canton (1996) I studied the optimal allocation of time across production and productivity-improving activity within a stochastic learning-or-doing model. This optimal allocation has been shown to depend on the state of the economy. Firms concentrate on production activity at the expense of research activity in good times when workers are highly productive. Research can better be done when times get worse. The opportunity costs of research - in terms of forgone production - are lower in a recession, so that it makes sense for firms to allocate more employees to the research department (or, in the terminology used here, send more employees to school). Another important difference with the analysis in Canton (1996) concerns the character of knowledge. In Canton (1996) it is assumed that firms can (at least partly) internalize the benefits from knowledge accumulation. They hence have an incentive to carry out research. In the present paper we consider knowledge as a purely public good. New knowledge immediately spreads out over the economy; markets for knowledge are completely missing. Without government action, no schooling activity would take place in such an economic setting.

There is a huge strand of literature related to the topics in this paper. Recent examples of studies on the effects of tax policies on long-term economic growth within a general equilibrium setting include Barro (1990), Barro and Sala-i-Martin (1995), Baxter and King (1993), Bovenberg and Smulders (1995), Glomm and Ravikumar (1994, 1997), Jones, Manuelli, and Rossi (1993), and Pecorino (1995). Barro (1990) introduces productive government services in the production technology available to firms, and shows that long-term economic growth is a hump-shaped function of the income tax rate needed to finance these public services. Barro and Sala-i-Martin (1995) allow for congestion externalities within the basic Barro-framework. Baxter and King (1993) study a number of fiscal policy experiments in a neoclassical model with elastic labour supply. Because of important interactions between capital and labour, public capital is shown to have important effects on private output and investment. Bovenberg and Smulders (1995) introduce productivity effects of a cleaner environment into the learning technology. Glomm and Ravikumar (1994, 1997) construct two-sector endogenous growth models in which the evolution of human capital in the learning sector is an increasing function of publicly provided "quality" of education. Jones, Manuelli, and Rossi (1993) focus on optimal taxation issues in a number of endogenous growth models, including a model that allows for government expenditures as a productive input in capital formation. In such a model, it is shown that the limiting capital tax rate is no longer zero. Pecorino (1995) investigates the relationship between tax rates and the present value of tax revenues in a two-sector endogenous growth model with endogenous labour supply. An increase in income taxation may lead to a reduction in tax revenues through labour supply and growth effects.

Examples of studies that allow for tax randomness include Ambler and Paquet (1994), Braun (1994), Christiano and Eichenbaum (1992), Jonsson and Klein (1996), and McGrattan (1994a,b). Ambler and Paquet (1994) introduce shocks to military expenditures in a real business cycle model where the government optimally chooses public investment and nonmilitary current expenditures. Fluctuations in personal and corporate income tax rates are included in a basic real business cycle model in Braun (1994). This is shown to improve the model's fit to the data, in particular with respect to employment variability. Christiano and Eichenbaum (1992) allow for government consumption shocks to influence labour market dynamics, also finding that this modification brings real business cycle theory more in line with the data. Jonsson and Klein (1996) investigate the role of fluctuations in distortive

(payroll and consumption) taxes and government consumption in explaining some of the salient features of the Swedish post-war business cycle experience. It is shown that models with stochastic fiscal policy perform statistically better than models without it. McGrattan (1994a,b) reestimates the contribution of technology shocks to the postwar U.S. business cycle experience in a basic real business cycle model extended with a government sector, fiscal disturbances, and tax randomness. Her findings suggest that something like 41% of the variance in output is explained by technology shocks, 28% by disturbances in government consumption, 27% in labour tax randomness, and 4% in capital tax shocks.

The remainder of the paper is structured along the following lines. In the next section we introduce the model. A calibration exercise is carried out in the third section. The model's implications for the relationship between long-term economic growth and income taxation are considered in section four. In section five the mechanics of the model are further investigated by examining transition dynamics following an imbalance between the stock of knowledge and physical capital, and impulse-response functions in the event of disturbances in total factor productivity, the marginal tax rate on capital income, and marginal taxation of labour income. Section six presents a quantitative assessment of the potential effects from tax randomness on business cycle variability. In section seven we evaluate the model's ability to replicate the post-war U.S. business cycle experience. Since employment fluctuations are too weak in the artificial economy compared to U.S. data, we introduce stochastic discounting in section eight. The model's implications for the interaction between economic growth and cyclical variability, and for the effect of government size on macroeconomic instability are discussed in section nine. Finally, section ten concludes the paper.

2. The model

In this section we construct a discrete time stochastic model of endogenous growth with learning-by-doing and learning-or-doing. There are three sectors in our economy: (i) households maximize their expected lifetime utility, (ii) firms maximize profits, and (iii) a government determines the tax rates on capital and labour as well as the composition of its expenditures on transfer payments and public schooling. All markets clear. We consider a decentralized economy: firms and households take prices as given. Also the actions undertaken by the government are taken as exogenously given; our analysis is positive rather

than normative.

Households

The economy is populated with a large number of identical infinitely-lived households. Each household maximizes expected lifetime utility, given by

$$U_0 = E \left[\sum_{t=0}^{\infty} \beta^t \{ \ln(C_t) - \zeta N_t \} \right] \quad (1)$$

where E is the expectation operator, $\beta < 1$ is the subjective discount factor, t is time, C is consumption, and N is labour supply. Abstracting from population growth, we interpret N as the fraction of the available time that the household allocates to market activity. The total time endowment is normalized to unity, so that $1-N$ is leisure time. ζ is a parameter that measures the relative weight that is attached to leisure time. Labour time decreases instantaneous utility in a linear fashion. This linearity assumption implies an infinite elasticity of substitution between leisure in different periods. The model can therefore be interpreted as a reduced form of the Hansen (1985) model with indivisible labour. Consumption enters into the utility function in logarithmic form: income and substitution effects cancel out and an interior solution is supported.

Households face the following resource constraint

$$K_t = R_t K_{t-1} + (1 - \tau_{N,t}) W_t N_t + \Pi_t - C_t \quad (2)$$

where K is the physical capital stock, R is the return on physical capital after taxation and depreciation, τ_N is a proportional tax on labour income, W is the wage rate, and Π denotes a lump-sum transfer payment from the government to the household.

Denoting the Lagrange multiplier of the resource constraint as λ , we obtain the following first order conditions:

$$1/C_t = \lambda_t \quad (3)$$

$$\zeta = \lambda_t (1 - \tau_{N,t}) W_t \quad (4)$$

$$1 = \beta E_t \left[\frac{C_t}{C_{t+1}} R_{t+1} \right] \quad (5)$$

These expressions have the usual interpretation. According to eq. 3, people consume up to the point where the marginal benefit of one additional unit of the consumption good equals the marginal cost in terms of decreased physical capital accumulation. Equation 4 says that, on the margin, one unit of time must be equally valuable in its two uses, leisure and market activity. Equation 5 is the Lucas asset pricing equation, after eliminating the Lagrange multiplier by substituting eq. 3 (*cf.* Lucas 1978). Following King and Rebelo (1988), we can interpret $\beta C_t/C_{t+1}$ as the modified discount factor.

Firms

Firms combine physical capital K , labour-augmenting knowledge Z , and production time L in order to produce one single homogeneous commodity. The production technology is concave with respect to physical capital K and effective labour input ZL separately, but exhibits constant returns to scale when factor inputs are accumulated at a uniform rate. Suppose that the production function is Cobb-Douglas:

$$Y_t = A_t K_{t-1}^\alpha [Z_{t-1} L_t]^{1-\alpha} \quad (6)$$

Y denotes aggregate output, and $0 < \alpha < 1$ ($1-\alpha$) is the production elasticity of physical capital (labour). Physical capital and knowledge are inherited from the past; their previous period's stocks show up in the current production technology. In line with real business cycle models, the exogenous productivity parameter A is assumed to be stochastic. More specific, the logarithm of A follows an AR(1)-process

$$\ln(A_{t+1}) = (1-\phi_A)\ln(A) + \phi_A \ln(A_t) + \epsilon_{A,t+1} \quad \epsilon_A \sim N(0, \sigma_A^2), \text{ i.i.d.} \quad (7)$$

The persistence of shocks to the production sector is denoted by ϕ_A . By restricting the persistence parameter to lie in the unit interval ($0 < \phi_A < 1$), we concentrate on temporary shocks only. Innovations are normally distributed with zero mean and a constant standard deviation σ_A . Competitive markets require

$$D_t = \alpha \frac{Y_t}{K_{t-1}} \quad (8)$$

$$W_t = (1 - \alpha) \frac{Y_t}{L_t} \quad (9)$$

D stands for dividend payments from the firms to the owners of the physical capital stock (the households). By definition, the relation between the return on physical capital after taxation and depreciation R and dividend payments D is given by

$$R_t = 1 + (1 - \tau_{K,t})(D_t - \delta) \quad (10)$$

where τ_K is a proportional tax on dividend income after deduction of capital depreciation, and $0 \leq \delta \leq 1$ is the depreciation rate of physical capital.

Knowledge accumulation

We shall assume that knowledge can be increased through learning-on-the-job, or learning-by-doing (see, for example, Arrow 1962, Romer 1986, Lucas 1988 & 1993, and Young 1991). Arrow (1962) and Romer (1986) assume that knowledge creation is a side product of investment in physical capital, whereas Young (1991) links learning to aggregate production activity. Here we assume that learning is the by-product of work experience and, as such, related to the time people spend on productive activity. In other words, we follow Lucas (1988) and relate learning to the fraction of hours worked. Secondly, we assume that employees not only learn by doing, but also from formal and informal schooling, and vocational training programs. In other words, knowledge accumulation is determined by unintentional learning-by-doing and intentional learning-or-doing. The learning technology is given by

$$Z_t = Z_{t-1}(\chi + \psi L_t + \omega v_t) \quad (11)$$

where v denotes the time spent on schooling activity. This specification allows for knowledge depreciation whenever χ is less than one. ψ is the transformation rate of production activity into knowledge formation, and ω is the transformation rate of time spent on schooling into

knowledge accumulation.² Labour market equilibrium requires

$$N_t = L_t + v_t \quad \forall t \quad (12)$$

Government

To close the model, we finally turn to a description of the government sector. Since knowledge is purely external to the individual household, the government fully reimburses the forgone earnings during schooling activity: the government finances these schooling activities by paying employees the current wage. Hours of schooling is therefore given by

$$v_t = \frac{G_t}{W_t} \quad (13)$$

where G is before-tax educational spending. It will be assumed that the government cannot issue debt claims (like, for example, in Chari, Christiano, and Kehoe 1993). The public sector has to stick to a simple balanced budget rule at each instant of time, *i.e.*

$$\Pi_t + (1 - \tau_{N,t})G_t = T_t = \alpha \tau_{K,t}Y_t + (1 - \alpha)\tau_{N,t}Y_t - \tau_{K,t}\delta K_{t-1} \equiv \tau_t Y_t - \tau_{K,t}\delta K_{t-1} \quad (14)$$

The sum of lump-sum transfers Π and after-tax educational spending $(1 - \tau_N)G$ must be equal to total tax revenues T . Since eq. 14 states that the government is required to finance current expenditures from current (distortionary) tax revenues, this balanced budget rule is a stylized version of the U.S. Gramm-Rudman-Hollings amendment of 1985 (see Baxter and King, 1993). From eqs. 8-10 it can be derived that capital tax revenues equal $\alpha \tau_{K,t}Y_t - \tau_{K,t}\delta K_{t-1}$, and labour tax revenues equal $(1 - \alpha)\tau_{N,t}Y_t$. The "composite" tax rate τ is given by $\alpha \tau_K + (1 - \alpha)\tau_N$.

Next we turn to the composition of government spending. We simply assume that a constant fraction η of total tax revenues is returned to the household in the form of lump-sum transfer payments, *i.e.*

$$\Pi_t = \eta(\tau_t Y_t - \tau_{K,t}\delta K_{t-1}) \quad (15)$$

The remaining part of the budget is used for public schooling, *i.e.*

² Notice that the basic RBC model abstracts from endogenous growth by implicitly assuming that $\psi = \omega = 0$. Exogenous growth can be allowed for by setting $\chi > 1$.

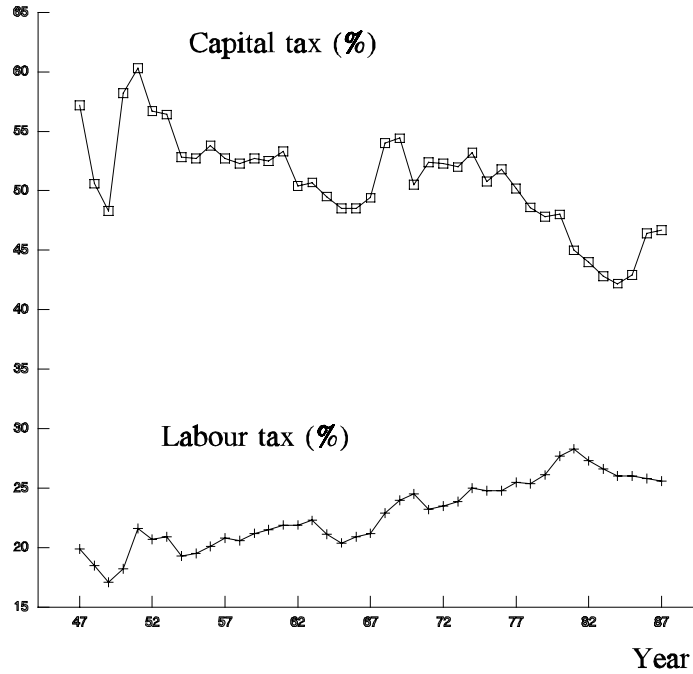
$$(1-\tau_{N,t})G_t = (1-\eta)(\tau_t Y_t - \tau_{K,t} \delta K_{t-1}) \quad (16)$$

Inspection of Figure 1 clearly illustrates that marginal tax rates on capital and labour have shown an erratic pattern through time. To allow for tax randomness in the analysis, let us assume that the time series properties of both tax rates can be described by independent AR(1)-processes:

$$\begin{bmatrix} \ln(\tau_{K,t+1}) \\ \ln(\tau_{N,t+1}) \end{bmatrix} = \begin{bmatrix} 1-\phi_K & 0 \\ 0 & 1-\phi_N \end{bmatrix} \begin{bmatrix} \ln(\tau_K) \\ \ln(\tau_N) \end{bmatrix} + \begin{bmatrix} \phi_K & 0 \\ 0 & \phi_N \end{bmatrix} \begin{bmatrix} \ln(\tau_{K,t}) \\ \ln(\tau_{N,t}) \end{bmatrix} + \begin{bmatrix} \epsilon_{\tau_K,t+1} \\ \epsilon_{\tau_N,t+1} \end{bmatrix} \quad (17)$$

$\epsilon_{\tau_K} \sim N(0, \sigma_K^2), \epsilon_{\tau_N} \sim N(0, \sigma_N^2), \text{ i.i.d.}$

Because the government is required to run a balanced budget, tax randomness will translate into randomness of government transfers to households and stochastic educational spending. Stochastic lump-sum government transfers can loosely be interpreted as aggregate demand disturbances, whereas an erratic pattern of public spending on schooling is an additional source of technology shocks.



Source: McGrattan (1994b)

Figure 1: Effective marginal tax rates on capital and labour in the U.S., 1947-1987.

Balanced growth

Next we shall determine the balanced growth path of the model. First we will define a competitive equilibrium.

Definition

For given realizations of the exogenous stochastic shocks $\{A_t, \tau_{K,t}, \tau_{N,t}\}_{t \in [0, \infty)}$, a *competitive equilibrium* is defined as a set of allocations $\{C_t, N_t, L_t, v_t, K_{t-1}, Z_{t-1}\}_{t \in [0, \infty)}$ and a set of prices $\{R_t, D_t, W_t\}_{t \in [0, \infty)}$ such that

- (i) $\{C_t, N_t\}_{t \in [0, \infty)}$ solve the representative household's problem,
- (ii) $\{L_t, K_{t-1}\}_{t \in [0, \infty)}$ solve the representative firm's problem,
- (iii) the resource constraint is given by eq. 2,
- (iv) the production technology is given by eq. 6,
- (v) the relation between R and D is given by eq. 10,
- (vi) the learning technology is given by eq. 11,
- (vii) the labour market clears, eq. 12,
- (viii) government spending is given by eq. 15 and 16.

It is important to realize that - in the terminology of Lucas (1988) - this competitive equilibrium is an *equilibrium* solution, but not necessarily an *optimal* solution selected by a social planner. Because of two external effects in our model economy - households do not internalize the learning effects into their labour supply decision and households do not take account of the beneficial effects of taxation in terms of increased productivity in private factor inputs - the equilibrium solution will be sub-optimal. Since our analysis is positive rather than normative, we leave a discussion of the social planner's solution for future work.

We transform the model into a stationary one by introducing some new variables. Let $c_t = C_t/K_{t-1}$, $y_t = Y_t/K_{t-1}$, $w_t = W_t/K_{t-1}$, $z_t = Z_t/K_t$, $\gamma_t^K = K_t/K_{t-1}$, $\gamma_t^Y = Y_t/Y_{t-1}$ denote the consumption-to-physical capital ratio, the production-to-physical capital ratio, the wage-to-physical capital ratio, the knowledge-to-physical capital ratio, the growth rate of physical capital, and the rate of economic growth. Along a balanced growth path it should hold that

$$\gamma^{K^*} = \gamma^{Y^*} = \gamma^*$$

Details on the solution procedure can be found in Appendix I. We turn in the next section to a calibration exercise to set the stage for an evaluation of the long-run growth effects from fiscal policy in section 4.

3. Calibration

In order to study the quantitative effects of government policies on economic growth and the business cycle, the model needs to be calibrated. The assumption of a Cobb-Douglas technology in the production sector implies that the production elasticity of physical capital, α , equals the capital share in national income. Following other RBC studies (Hansen 1985, for instance), we set $\alpha=0.36$. The discount factor β is set equal to 0.96, interpreting one period to correspond to a year (*cf.* Kydland and Prescott 1982). Physical capital depreciation is set at 6% annually (*cf.* Stokey and Rebelo, 1995). Estimates of the annual rate of human capital (or knowledge) depreciation vary widely; Stokey and Rebelo (1995) calculate human capital depreciation to lie in the range of 2.7-8.0%. We set the rate of knowledge depreciation at 4%. The scaling parameter A is set at 1. McGrattan (1994a,b) reports annual data on effective marginal tax rates on capital and labour for the U.S. 1947-87 period. We use her average values in our benchmark calibration: 0.51 for capital taxes and 0.23 for labour taxes. Following Jones, Manuelli, and Rossi (1993), it will be assumed that the employed spend about 5% of their available time in schooling activity ($v=0.05$). ζ is calibrated at 2.32 so that the model predicts labour supply to be 0.3, which corresponds to empirical observations on average hours worked (see, again, Kydland and Prescott 1982, and Hansen 1985). To replicate these observations, we pick $\eta=0.64$; 36% of government revenues is used for educational spending, whereas the remaining 64% is rebated to households in the form of lump-sum transfer payments. In order to replicate an average annual growth in per capita income of 2% for the U.S., we set the transformation rate of production time into learning (ψ) at 0.12, in combination with a transformation rate of schooling activity into learning (ω) of 0.6: it is thereby assumed that public schooling and learning-by-doing contribute equally to knowledge formation.

Finally we want to select reasonable parameters for the exogenous stochastic processes. In McGrattan (1994a,b) it is calculated that $\check{\phi}_K=0.976$; $\check{\sigma}_K=0.0108$; $\check{\phi}_N=0.970$; $\check{\sigma}_N=0.0034$ (a breve is used to distinguish parameters pertaining to quarterly figures from those pertaining to annual figures). These parameter estimates are transformed to their annual counterparts by using standard formulas.³ Following the RBC literature, we set the

³ Namely: $\phi=\check{\phi}^4$ and $\sigma^2=\check{\sigma}^2(1+\phi^2+\phi^4+\phi^6)\approx\check{\sigma}^4$, or $\sigma\approx 2\check{\sigma}$.

persistence parameter for the AR(1)-process generating shocks to the production sector at 0.81, which corresponds to a commonly used value of 0.95 for quarterly series. The model approximately replicates the empirically observed variability of aggregate output by setting the standard deviation of the innovation term at 0.0153. Table 1 summarizes the calibration exercise.

Production elasticity of physical capital	α	0.36
Discount factor	β	0.96
Depreciation of physical capital	δ	0.06
Leisure parameter	ζ	2.32
Allocation parameter	η	0.64
Tax on capital income	τ_K	0.51
Tax on labour income	τ_N	0.23
Depreciation of knowledge	$1-\chi$	0.04
Learning-by-doing	ψ	0.12
Learning-or-doing	ω	0.6
Productivity parameter (unconditional mean)	A	1
Persistence of shocks to productivity parameter	ϕ_A	0.81
Innovation term	σ_A	0.0153
Persistence of shocks to capital taxes	ϕ_K	0.9074
Innovation term	σ_K	0.0189
Persistence of shocks to labour taxes	ϕ_N	0.8853
Innovation term	σ_N	0.0058

Table 1: Baseline parameter values.

4. Fiscal policy and economic growth

In order to analyze the consequences of a permanent change in the capital tax rate and the labour tax rate for the long-term rate of economic growth, we undertake two policy experiments in this section. In the first experiment we vary the proportional tax on capital income, holding the labour tax constant. The second experiment involves a change in the labour income tax, keeping the capital taxation rate fixed. For both of these experiments, we explore the economy's long-run dynamic response to such policy changes.

Variation in distortionary capital taxation

In this experiment we vary the capital income tax rate between 0 and 0.8, while keeping the tax on labour income constant at 0.23.

Figure 2 shows that economic growth is an *increasing* function of the capital tax rate for the baseline parameter constellation, but may become a decreasing function for other parameter choices. Three effects need to be distinguished to understand this result. Capital taxation has repercussions on schooling activity, on production time, and on labour supply. An increase in capital taxes will go along with higher educational spending; this will increase economic growth. However, for given labour supply, this increased schooling activity will crowd-out production activity so that learning-by-doing will decline. This may reduce economic growth. The interaction between capital taxation and labour supply is ambiguous, and depends on the composition of government spending. The intuition of this result is the following. Since knowledge is purely external to the household, any public spending to increase knowledge formation is considered by the household as being wasteful. In other words, households think of the government as some institution that collects taxes and throws part of the revenues into the ocean. Therefore, to the extent that the receipts are not transferred back to the household, taxation is considered by the household as a direct income loss. The negative wealth effect associated with this income loss will induce agents to decrease their demand for leisure, *i.e.* to increase their labour supply. So capital taxation may increase labour supply when this income effect is large enough (*i.e.* when $1-\eta$ is large enough).

In our benchmark parameter constellation it has been assumed that η is relatively high

(so that the negative wealth effect is relatively weak), and public schooling is important in the learning technology. Capital taxation induces people to decrease their labour supply, which adversely affects long-term economic growth through the reduction in learning-by-doing. Capital taxation is used to finance public schooling that contributes favourably to long-term economic growth, but reduces the scope for learning-by-doing. Overall, the positive growth effect from increased schooling activity dominates the adverse growth effect from reduced labour supply and decreased learning-by-doing: capital taxation increases economic growth in the investigated domain.⁴

Two other scenarios are investigated. Firstly, we decrease η to 0.4 so that the negative wealth effect associated with educational spending is increased. Indeed, as Figure 2c illustrates, this perceived income loss will induce agents to decrease their demand for leisure, *i.e.* to increase their labour supply. The crowding-out effect from schooling time on production activity is thereby reduced, and the positive relationship between capital taxation and economic growth is even more pronounced. Secondly, we decrease ω to 0.1 and increase ψ to 0.2, *i.e.* learning-by-doing becomes more important in the learning technology, and schooling becomes less important. Capital taxation now has adverse effects on economic growth. The positive effect from increased schooling is weaker than the negative effect from reduced learning-by-doing.

⁴ Such a potential positive relationship between long-term economic growth and capital taxation has been investigated in a number of other studies. For instance, Uhlig and Yanagawa (1996) construct an OLG-model in which taxation of capital income accruing to the old may relieve the tax burden on the young. In turn, this could lead to increased savings and economic growth. Another example of this literature is Smith (1996).

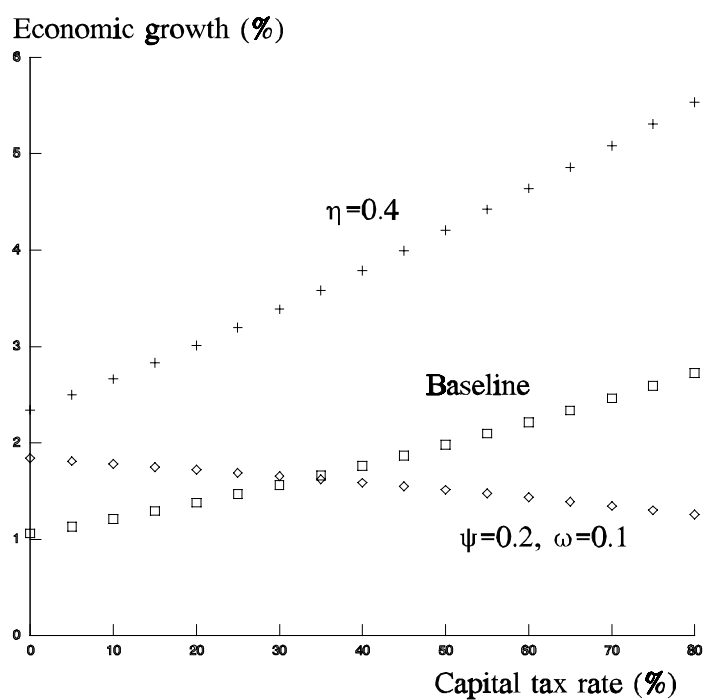


Figure 2a: Capital taxation and economic growth.

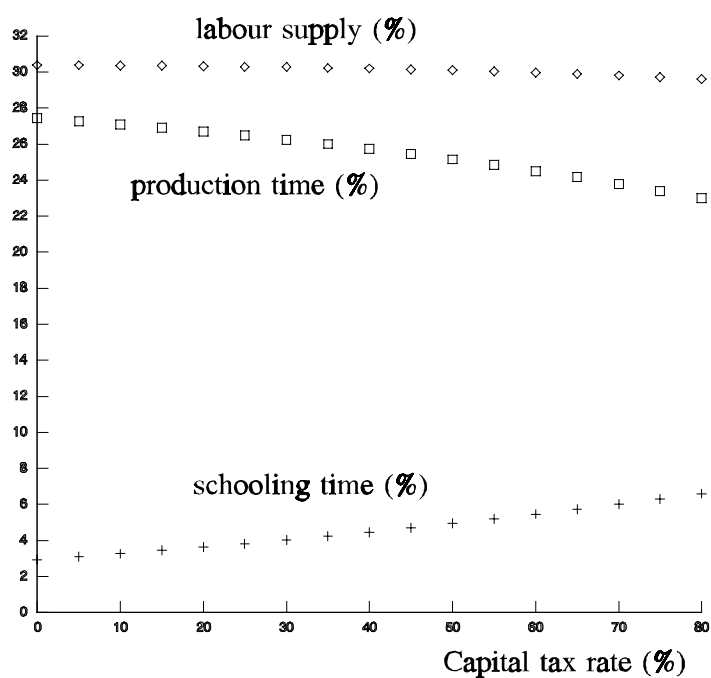
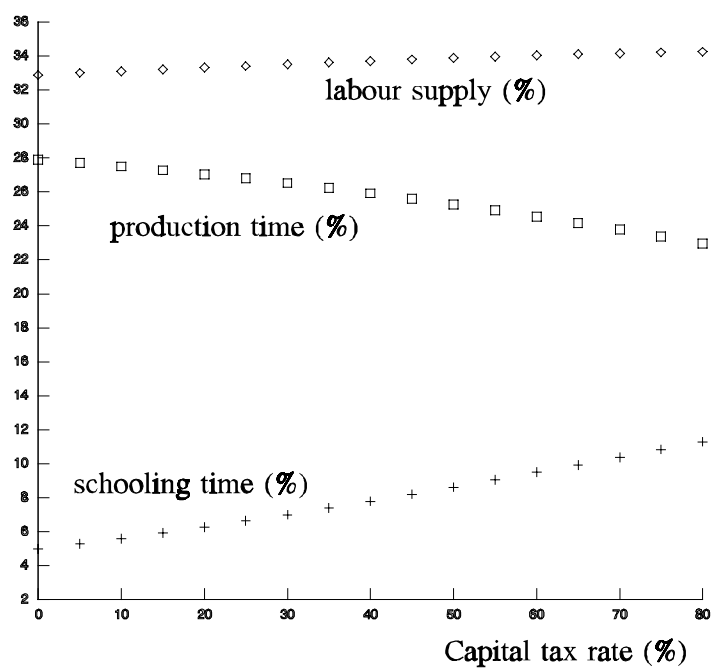
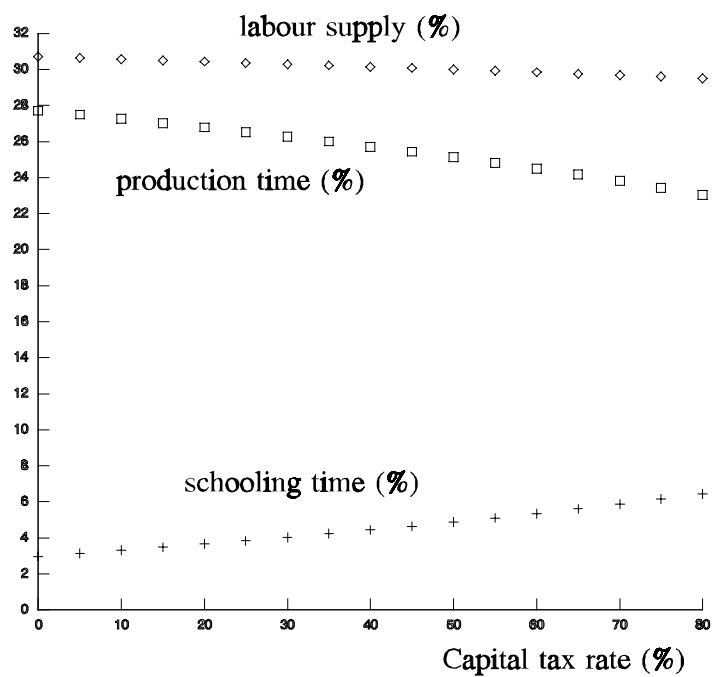


Figure 2b: Capital taxation and the labour market, baseline.

Figure 2c: Capital taxation and the labour market, $\eta=0.4$.Figure 2d: Capital taxation and the labour market, $\psi=0.2$ and $\omega=0.1$.

Variation in distortionary labour taxation

In the second experiment we keep the capital tax rate constant at 0.51 and vary labour income taxation between 0 and 0.8.

Figure 3a illustrates the quantitative findings. As before, there are three effects going on. On the one hand, the tax on labour induces households to shift to non-taxable leisure time, so that labour supply will decline. Revenues from labour taxation are partly used for public schooling, which increases economic growth. Thirdly, the reduction in labour supply and the increase in schooling activity crowd-out production activity, and thereby the fraction of knowledge accumulation attributable to learning-by-doing. The overall effect on economic growth will depend on the effectiveness of schooling as an input in the knowledge technology relative to learning-by-doing, and on the importance of the negative wealth effect. A positive relationship between economic growth and labour taxation is found for the baseline parameter constellation. The growth-enhancing effect of increased schooling activity exceeds the negative effect on economic growth caused by the decline in production time.

Different relationships between long-term growth and labour taxation may exist for other parameter sets. For instance, when η is decreased to 0.4 so that the negative wealth effect associated with educational spending is increased, we find an even stronger positive effect from labour taxation on economic growth. The intuition is that the negative effect from labour taxation on labour supply (substitution to non-taxable leisure activity) is mitigated by the wealth effect inducing agents to decrease their demand for leisure. Secondly, we decrease ω to 0.1 and increase ψ to 0.2, *i.e.* learning-by-doing becomes more important in the learning technology, and schooling becomes less important. Labour taxation now has adverse effects on economic growth. The positive effect from increased schooling is weaker than the negative effect from reduced learning-by-doing.

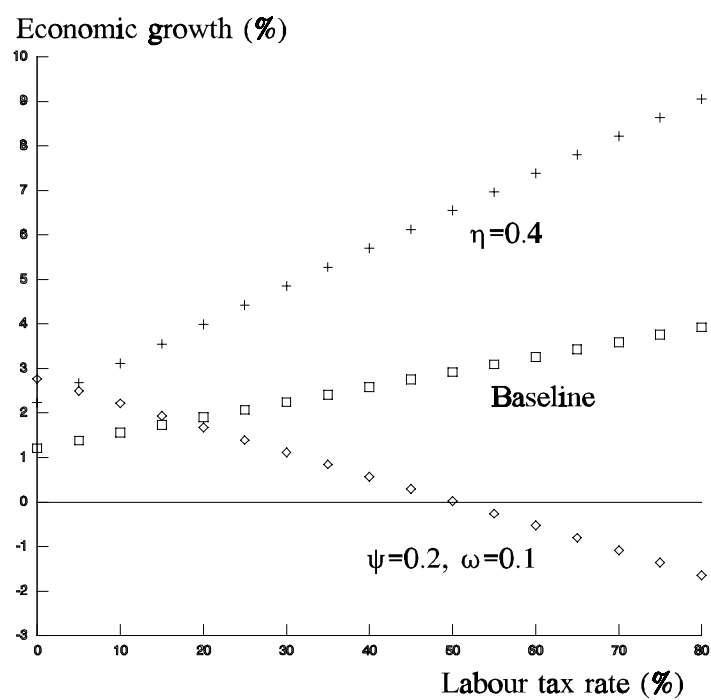


Figure 3a: Labour taxation and economic growth.

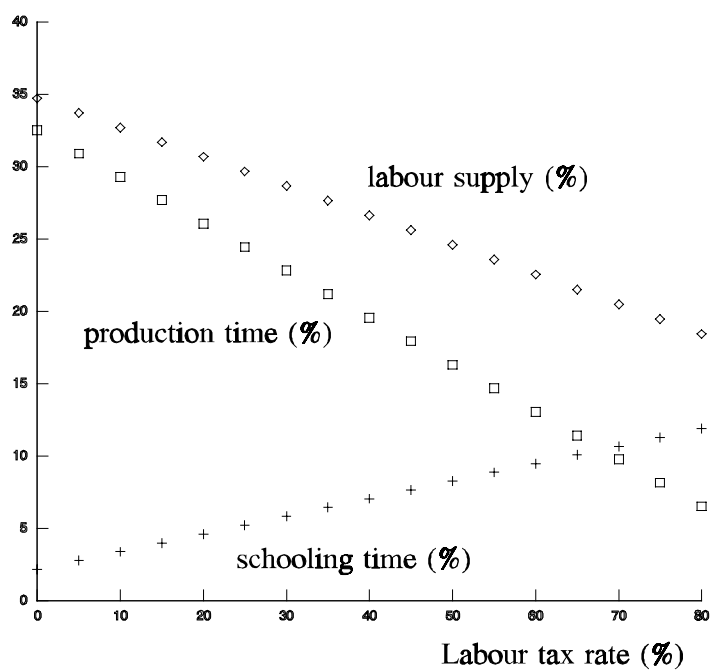


Figure 3b: Labour taxation and the labour market, baseline.

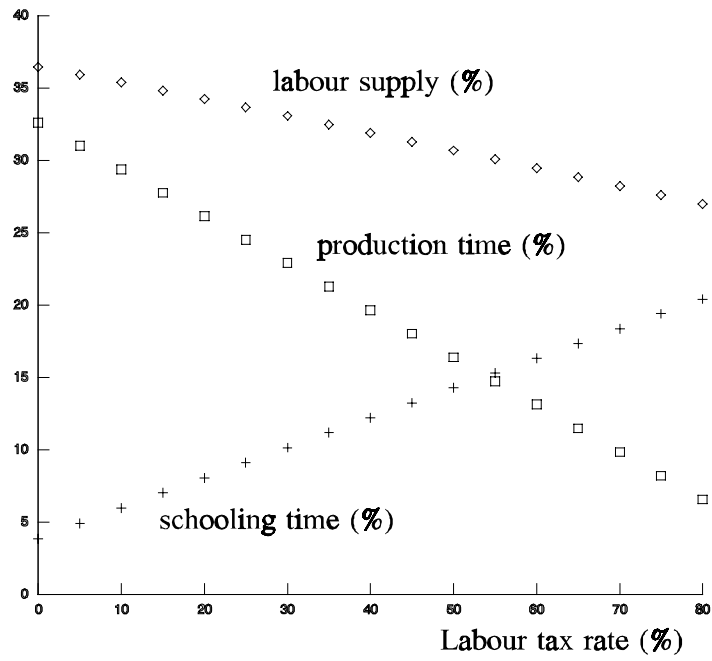


Figure 3c: Labour taxation and the labour market, $\eta=0.4$.

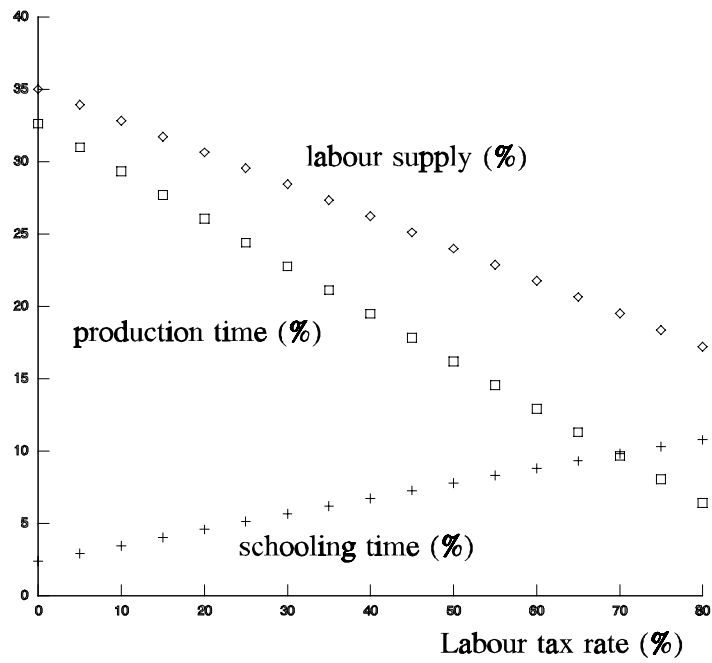


Figure 3d: Labour taxation and the labour market, $\psi=0.2$ and $\omega=0.1$.

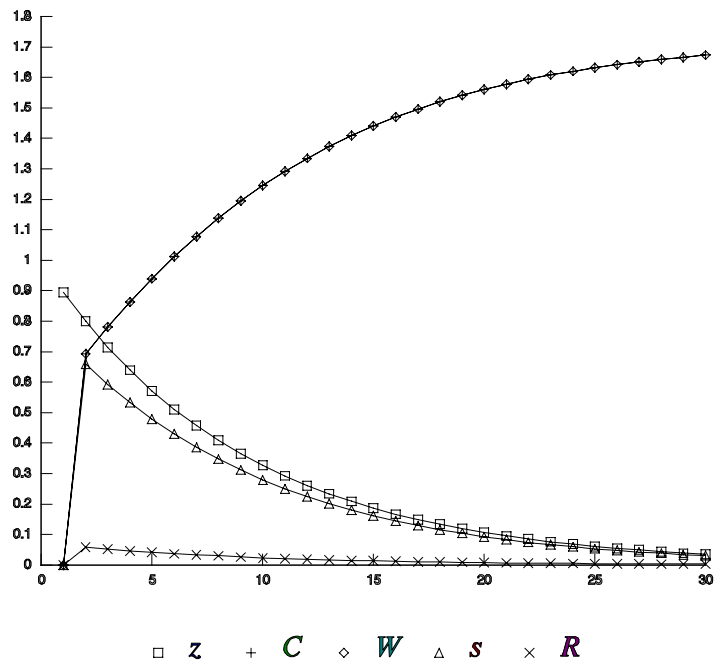
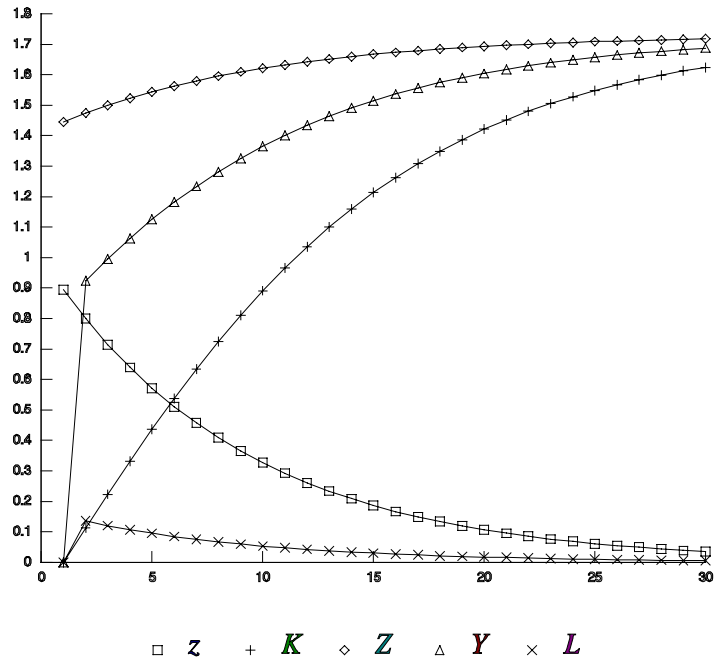
5. Transition dynamics and impulse-response functions

In order to understand the mechanics of our model, we will concentrate in this section on transition dynamical adjustment trajectories and impulse-response analysis: how does the economic system respond to a disturbance in one of the variables describing the state of the economy? Computational details can be found in Appendix II.

The economy's transition dynamics is shown in Figure 4. This figure illustrates the system's dynamics when the initial knowledge-to-physical capital ratio is 1% above its balanced growth value. Such a relative abundance of knowledge increases the rental rate on physical capital, so that saving and physical capital accumulation are encouraged. We find that the initial capital imbalance gradually disappears and the knowledge-to-physical capital ratio converges to its balanced growth value z^* . Consumption and wages permanently increase as the restoration of the knowledge-to-physical capital ratio makes it possible to raise production. The substitution effect associated with the increase in the wage rate tends to encourage labour supply and production activity in the early stage of the adjustment process, but this effect is eventually counterbalanced by the income effect stemming from the increase in production which tends to raise the demand for leisure.

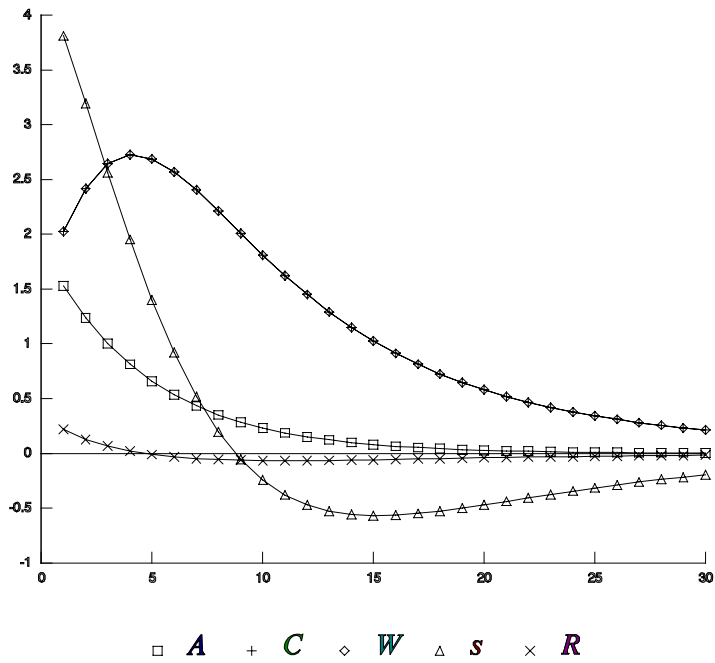
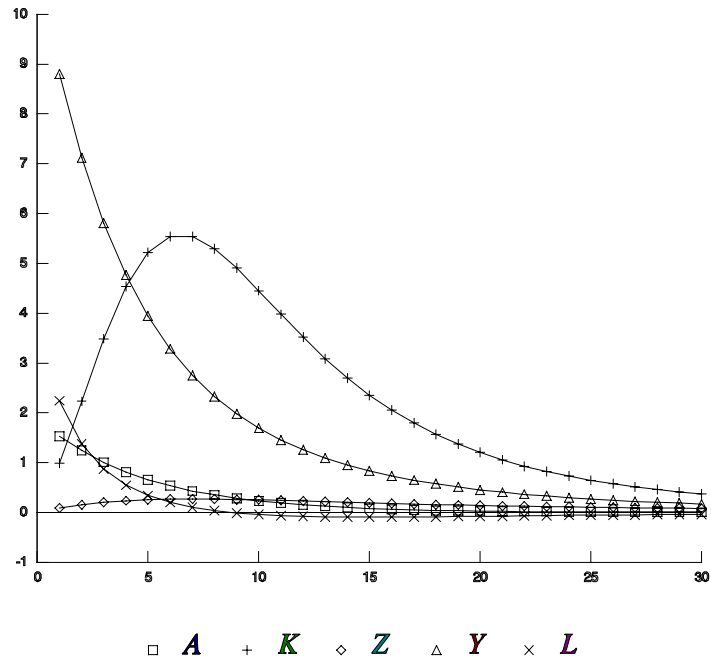
In Figure 5 we plot the impulse-response function when the economic system is hit by a positive one standard deviation productivity shock. The temporary increase in productivity induces households to increase their labour supply, since wage rates have gone up, leading to an increase in knowledge accumulation (learning-by-doing), and output. The increase in production exceeds the increase in consumption (agents smooth consumption possibilities intertemporally), so that more physical capital is accumulated. Finally, the marginal product of physical capital increases, and the rental rate is raised in the short-term.

In Figure 6 the impulse-response functions are drawn when the economic system is hit by a positive one standard deviation capital taxation shock. The temporary capital tax increase reduces households' labour supply and savings in the short term, and crowds-out production time since schooling activity is intensified. Ultimately, knowledge accumulation is increased in this example: the positive effect from increased public schooling on knowledge formation is larger than the negative effect from reduced learning-by-doing. Although physical capital accumulation is slowed-down in the short term, interactions between knowledge and physical capital eventually increase the physical capital stock.



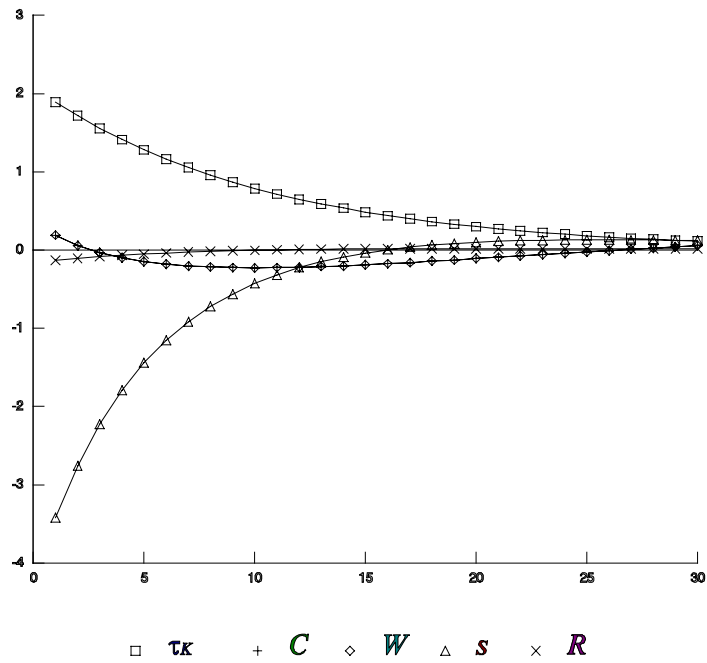
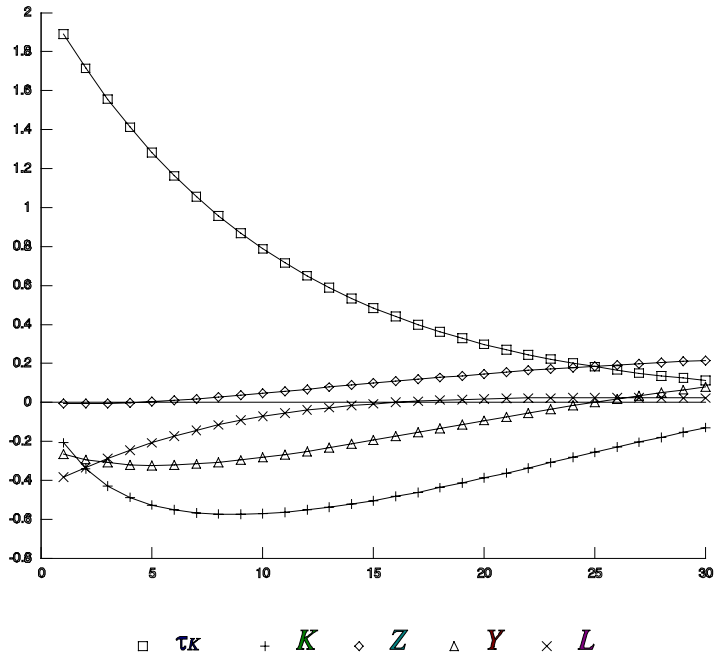
Note: The vertical axis measures %-deviations from the balanced growth path; time is on the horizontal axis.

Figure 4: Transition dynamics.



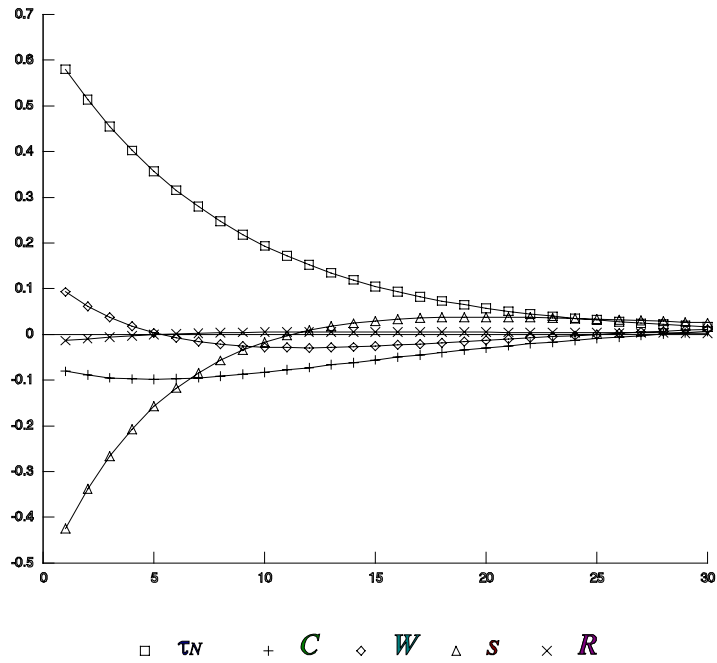
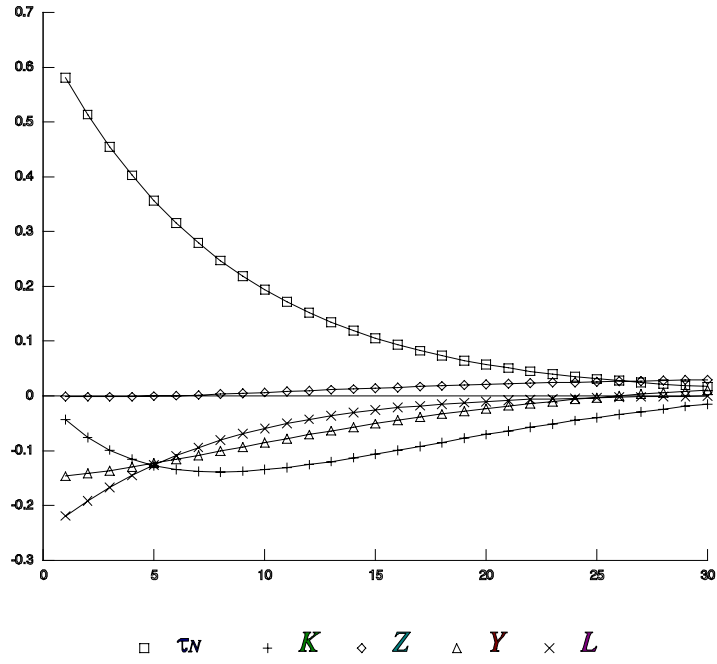
Note: The vertical axis measures %-deviations from the balanced growth path; time is on the horizontal axis.

Figure 5: Impulse-response functions in case of a technology shock.



Note: The vertical axis measures %-deviations from the balanced growth path; time is on the horizontal axis.

Figure 6: Impulse-response functions in case of a capital tax shock.



Note: The vertical axis measures %-deviations from the balanced growth path; time is on the horizontal axis.

Figure 7: Impulse-response functions in case of a labour tax shock.

The impulse-response functions in case that the economy is perturbed by a one standard deviation labour tax increase are illustrated in Figure 7. The temporary labour tax increase reduces households' labour supply and savings in the short term. Again, knowledge accumulation is increased in this example: the positive effect from increased schooling activity on knowledge formation is larger than the negative effect from reduced learning-by-doing. Although physical capital accumulation is slowed-down in the short term, interactions between knowledge and physical capital again increase the physical capital stock as the adjustment process continues.

6. Tax randomness as an additional source of business cycle fluctuations; A quantitative assessment

In this section we turn to a quantitative evaluation of tax randomness as an additional source of cyclical variability. Stochastic income taxation translates into randomness on the government's expenditure side under a balanced budget regime. It has been assumed that the composition of government spending is constant, implying that tax randomness translates into stochasticity in transfer payments (loosely interpreted as "demand shocks") as well as in educational government spending (loosely interpreted as "technology shocks").

In Table 2 we present the results. Decomposing the standard deviation of the artificially generated time series for output, consumption, investment, employment, and physical capital into the separate contributions from the underlying exogenous stochastic processes reveals that tax randomness contributes modestly to the business cycle pattern. About 7% of output variability is due to stochastic tax rates in this model economy; 3.3% is explained by random capital taxes and 3.8% by stochastic labour taxation. The effect of tax stochasticity on cyclical variability differs substantially across the series. Randomness in capital taxation accounts for 10% of the investment cycle, and for about 9% of variability in the physical capital stock. Innovations in labour taxes are particularly important for physical capital fluctuations.

Series	% standard deviation explained by innovations in		
	A	τ_K	τ_N
Model economy			
Output	92.9	3.3	3.8
Consumption	90.4	4.2	5.3
Savings	87.6	10.1	2.4
Employment	86.2	9.9	3.9
Physical capital	83.5	9.3	7.2

Note: Simulated series are logged and detrended by the Hodrick-Prescott filtering technique, setting the smoothing parameter at 400 (a common choice for annual data). The artificially generated second moments are averages across 200 runs of 40 periods.

Table 2: Decomposition of cyclical variability.

McGrattan (1994a,b) reestimates the contribution of technology shocks to the postwar U.S. business cycle experience in a basic real business cycle model extended with a government sector, fiscal disturbances, and tax randomness. Her findings suggest that something like 41% of the variance in output is explained by technology shocks, 28% by disturbances in government consumption, 27% in labour tax randomness, and 4% in capital tax shocks. We obtain similar results for capital tax stochasticity (explaining 3.3% of output variability), but find much smaller effects of labour tax fluctuations (3.8% of output fluctuations). This difference could be due to the fact that we adopt a different utility function (with an infinite elasticity of substitution between leisure in different periods) than McGrattan.

7. Bringing the model to the data

After having evaluated the cyclical implications of tax randomness in the previous section, we now turn to the crucial questions: is our model a reasonable business cycle model, and does tax randomness improve the model's performance? To put it differently, how well does our model replicate some salient business cycle facts, and what is the contribution of random taxation? Table 3 summarizes some business cycle properties for different variants of the model, and compares these numbers with those observed for the U.S. economy. t -tests are carried out in order to check whether the artificially generated numbers differ significantly

from the actual data, the null hypothesis being that the generated number equals the actual number. An asterisk denotes that the null hypothesis cannot be rejected at conventional significance intervals.

Compared to the artificial economy with constant tax rates (the second column in Table 3), the introduction of tax randomness improves the model's fit along various lines. Whereas the economy with constant tax rates cannot replicate variability in physical capital correctly (more precisely, the standard deviation of logged and detrended physical capital does not significantly deviate from its observed number in the U.S. economy at conventional significance levels), a model with tax randomness can account for observed capital stock variability. With respect to predicted correlations with output, a model that allows for tax randomness can explain the correlation between employment and output correctly, whereas an artificial economy without stochastic taxation fails to reproduce any of the observed correlations with output correctly. By-and-large (and despite of a few failures), the stochastic endogenous growth model with stochastic taxation does a better job than the same model without tax randomness. Our model seems to give a close description of some salient U.S. business cycle characteristics: the model economy can explain simultaneously observed cyclical variability in production, consumption, savings, employment, and physical capital, as well as observed cross-correlations of output with employment. Without claiming too much, our preliminary econometric evaluation at least suggests that the model constructed in this paper can compete successfully with existing real business cycle models.

	U.S. economy	Artificial economy, constant tax rates	Artificial economy, stochastic tax rates	Artificial economy, stochastic tax rates, and stochastic discounting
σ_Y	2.45	2.45* (0.42)	2.44* (0.47)	2.44* (0.43)
σ_C	1.80	1.35* (0.27)	1.37* (0.32)	1.51* (0.34)
σ_s	8.32	6.91* (1.12)	7.29* (1.21)	6.85* (1.05)
σ_L	1.60	1.25* (0.20)	1.34* (0.22)	1.79* (0.31)
σ_K	2.42	1.50 (0.40)	1.56* (0.45)	2.05* (0.59)
ρ_C	0.87	0.94 (0.01)	0.91 (0.03)	0.68* (0.11)
ρ_s	0.78	0.93 (0.02)	0.90 (0.04)	0.77* (0.12)
ρ_L	0.86	0.93 (0.02)	0.89* (0.04)	0.78* (0.07)
ρ_K	0.39	0.65 (0.04)	0.64 (0.07)	0.65 (0.11)
J		1,0549 $\times 10^5$	111.6	8.8

Note: $s=(Y-C)/Y$ is the savings rate. Variables are logged and detrended by the Hodrick-Prescott filtering technique, setting the smoothing parameter at 400 (a common choice for annual data). Summary statistics for the U.S. economy are taken from Einarsson and Marquis (1994, Table 1). The artificially generated second moments σ and correlation figures with output ρ are averages across 200 runs of 40 periods. σ_A is adjusted in order to replicate the standard deviation of aggregate production in the U.S.: $\sigma_A=0.0157$ for the artificial economy with constant tax rates; $\sigma_A=0.0153$ for the artificial economy with stochastic tax rates; $\sigma_A=0.014$ for the artificial economy with stochastic tax rates and stochastic discounting. In the latter economy we used $\phi_\beta=0.95$ and $\sigma_\beta=0.003$. An asterisk indicates that the null hypothesis that the number in the artificial economy equals the observed number for the U.S. economy, cannot be rejected at conventional significance levels. Standard errors are in parentheses.

Table 3: A confrontation of the models with U.S. statistics.

An assessment of the model in terms of the individual time series properties suggests that the inclusion of tax randomness brings the artificial economy into closer conformity with the data. Let us now proceed by investigating whether the model can pass the more rigorous

Wald-test. The actual and simulated series are respectively given by

$$\begin{aligned} \{X_t\}_{t=1}^T, & \quad X \in \{Y, C, s, L, K\} \\ \{\hat{X}_t^i\}_{t=1}^T, & \quad \hat{X} \in \{\hat{Y}, \hat{C}, \hat{s}, \hat{L}, \hat{K}\} \end{aligned}$$

where $i=1, \dots, N$ denotes the simulation. The U.S. series from Einarsson and Marquis (1994) that were used for X_t cover the 1950-89 period, so that $T=40$. The simulated series \hat{X}_t^i also cover 40 time units. The null hypothesis is

$$H_0: \begin{pmatrix} \sigma_Y \\ \vdots \\ \sigma_K \end{pmatrix} = \begin{pmatrix} \sigma_{\hat{Y}} \\ \vdots \\ \sigma_{\hat{K}} \end{pmatrix}$$

We thus test the model's ability to reproduce the observed second moments for a number of series simultaneously. The relevant test-statistic is given by

$$J = \left[\begin{pmatrix} \sigma_Y \\ \vdots \\ \sigma_K \end{pmatrix} - \begin{pmatrix} \sigma_{\hat{Y}} \\ \vdots \\ \sigma_{\hat{K}} \end{pmatrix} \right]' M \left[\begin{pmatrix} \sigma_Y \\ \vdots \\ \sigma_K \end{pmatrix} - \begin{pmatrix} \sigma_{\hat{Y}} \\ \vdots \\ \sigma_{\hat{K}} \end{pmatrix} \right] \rightarrow \chi^2(k) \text{ (under } H_0)$$

where k is the dimension of the vector containing the second moments. The weight matrix M is approximated by the simulated version:

$$\hat{M} = \left[\frac{1}{N} \sum_{i=1}^N \begin{pmatrix} \sigma_{\hat{Y}^i} \\ \vdots \\ \sigma_{\hat{K}^i} \end{pmatrix} (\sigma_{\hat{Y}^i} \dots \sigma_{\hat{K}^i}) \right]^{-1}$$

where $i=1, \dots, N$ denotes the simulation, and T is the length of the actual and simulated series. The J -statistics are reported in the last row in Table 3. The null hypothesis is dramatically rejected in an economy with constant tax rates. A model which includes stochastic taxation is still rejected, but the J -statistic is sharply reduced.

8. Stochastic discounting

Although the introduction of tax randomness in combination with an infinite elasticity of substitution between leisure in different periods goes a long way toward explaining observed employment fluctuations, still a considerable fraction of employment swings cannot be mimicked: for realistic output variability our model can (on average) explain about 84% ($=1.34/1.6$) of U.S. employment volatility. This inadequacy is largely responsible for the models' failure to pass the Wald-test. It is a common shortcoming of real business cycle models, known in the literature as the employment variability puzzle. It also turned out that movements in consumption and physical capital are too smooth in the simulated series. To try and solve these puzzles, we inevitably have to resort to the introduction of an additional source of stochasticity. Let us assume that the logarithm of β follows an AR(1)-process⁵

$$\ln(\beta_{t+1}) = (1-\phi_\beta)\ln(\beta) + \phi_\beta\ln(\beta_t) + \epsilon_{\beta,t+1} \quad \epsilon_\beta \sim N(0, \sigma_\beta^2), \text{ i.i.d.} \quad (18)$$

The persistence of shocks to the discount factor is denoted by ϕ_β , and innovations are normally distributed with zero mean and a constant standard deviation σ_β . These stochastic preferences can be interpreted as waves in consumers' trust in the economy or "animal spirits" regarding consumption and leisure opportunities. Since we expect preferences to change slowly, we assume that preference shocks are more persistent than technology and tax perturbations. In the calibration exercise we pick $\phi_\beta=0.95$, and $\sigma_\beta=0.003$. The last column in Table 3 presents the results.

Labour market fluctuations are now exaggerated in the theoretical economy, but the model now gives a better description of consumption and physical capital variations. The artificial economy can also explain observed cross-correlations of output with consumption, savings, and employment. Let us now go back to the Wald-test described in the previous section. The J -statistic equals 8.8, which is safely below the critical value, even at a 5% significance level. Winding up this section, to bring the model more in line with the data we proposed to introduce stochastic discounting. Our findings imply that a theoretical economy

⁵ In an earlier version of this paper we experimented with stochastic preferences for leisure. We thus were able to replicate observed employment movements, but consumption variability and physical capital fluctuations were again underestimated. We therefore decided to explore the model's characteristics in case of stochastic discounting.

may pass the Wald-test successfully.

9. Discussion

Interaction between long-run growth and cyclical variability

One question in this paper is whether long-term economic growth and cyclical variability are interdependent. Particularly, does an increase in the intensity of business cycle fluctuations generally go along with higher or lower economic growth? Until now, empirical research has not provided a satisfactory answer to the question whether economic growth and the business cycle are interrelated. The number of studies on this topic is limited, and the results are mixed. Some authors find a significant positive effect of cyclical variability on economic growth (*cf.* Kormendi and Meguire 1985, Grier and Tullock 1989), while others conclude the opposite (*cf.* Ramey and Ramey 1995, Martin and Rogers 1995). These studies typically carry out cross-country regressions of growth rates on a number of explanatory variables, including some measurement of business cycle variability. As Levine and Renelt (1992) have convincingly shown, the outcome of the statistical analysis crucially depends on the set of control variables. Levine and Renelt conclude that cyclical variability and economic growth are not related in a robust manner.

Another interpretation of the different results is that the regression analysis suffers from an endogeneity problem, making the results spurious.⁶ Such an endogeneity bias exists when economic growth and business cycle variability are simultaneously affected by some other factor. For instance, one can think of political stability as a factor that might influence both the growth performance and the stability of this growth process: countries with unstable government coalitions may suffer lower growth and more cyclical variability than nations with a more stable political climate. The openness of a country is another candidate for factors that impact on growth and business cycles simultaneously: nations that trade a large share of their GDP on world markets may enjoy faster economic growth and less business cycle variability through international risk-sharing. A third potential explanation for economic growth and cyclical fluctuations is government policy. This is the factor we focused on in the present

⁶ I thank José-Víctor Ríos-Rull for discussing this point with me.

paper. Particular attention is directed at the role of labour and capital taxation on economic growth and cyclical variability. To evaluate the model's implications for the interdependence between long-run growth and the cycle, we carry out two experiments. In the first experiment we vary the unconditional mean of the labour tax rate, holding constant the unconditional mean of the capital tax rate and the standard deviation of innovations in technology and taxation. As shown earlier, labour taxation can go along with higher or lower economic growth, depending on the specific set of parameters. We did not find a clear relationship between labour taxation and output variability in this experiment. Results are therefore not reported. Secondly, we vary the unconditional mean of the capital tax rate, holding constant the unconditional mean of the labour tax rate and the standard deviation of innovations in technology and taxation. Results are shown in Figure 8 and 9. Again, capital taxation can increase or decrease economic growth. Capital taxes spur economic growth (in the investigated domain) for the baseline parameter set, but slow-down economic growth when, for example, learning-by-doing becomes more important in the learning technology, and schooling becomes less important (*cf.* Figure 2a). The figures also show that increased capital taxation reduces output variability.⁷ This reduction in macroeconomic instability is basically due to smoother employment fluctuations. By-and-large, decreased cyclical variability may go along with higher or lower economic growth as capital taxation increases. This may cause spuriousness in regression studies in which growth rates are regressed against some measurement of cyclical variability, without controlling for differences in the tax structure.

⁷ HP-filtered output variability is derived from experiments with $\tau_k=0; 10\%; 20\%; 30\%; 40\%; 50\%; 60\%; 70\%$, holding τ_N constant at 23%. The experiments (200 runs of 40 periods) are repeated three times.

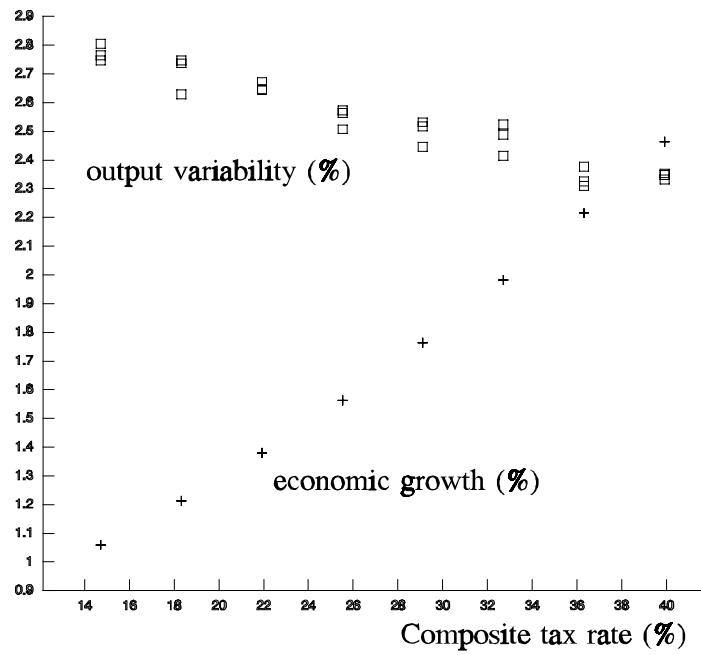


Figure 8: "Negative interaction" between economic growth and the business cycle.

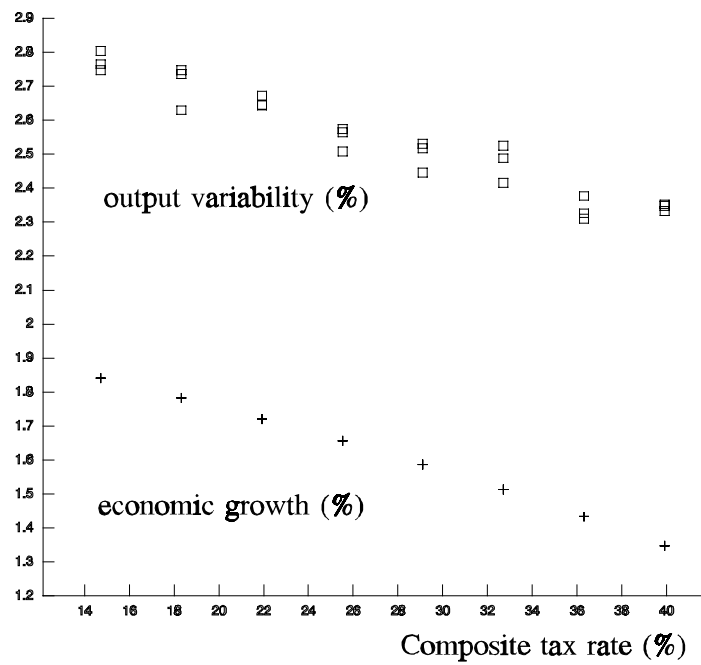
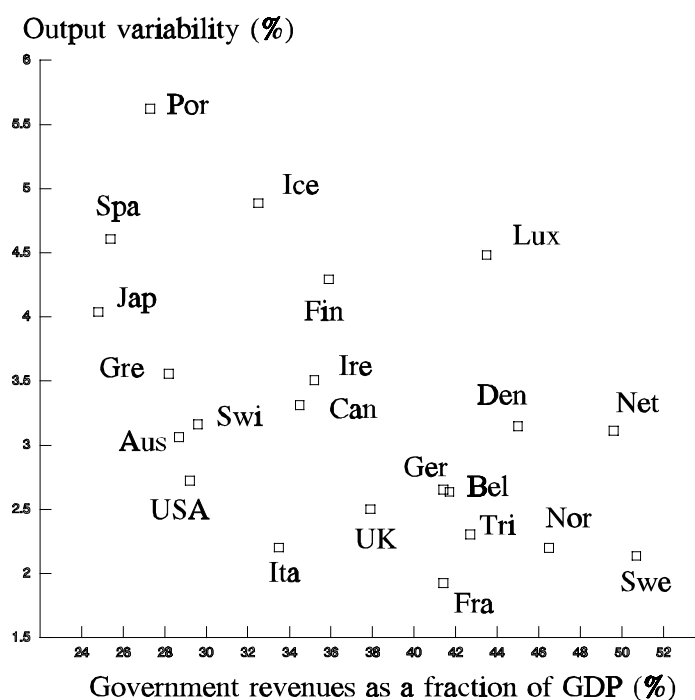


Figure 9: "Positive interaction" between economic growth and the business cycle.

Government size and macroeconomic instability

Another interesting question is whether the size of the government matters for the intensity of business cycle fluctuations or not. This question at least goes back to the Keynesian literature on "automatic stabilizers" (see, for instance, Baily (1978), DeLong and Summers (1986)). More recently, Galí (1994) finds that macroeconomic instability tends to decrease when governments become larger for 22 OECD countries. For instance, a 10%-point increase in the government revenues/GDP ratio leads to a 1.6%-point reduction of output variability (the standard deviation of per capita GDP from trend). Galí (1994) does not pass the raw output data through the Hodrick-Prescott (HP) filter, so that output fluctuations may be exaggerated. We therefore applied the HP-filtering technique to per capita income data (obtained from Summers and Heston PWT 5.6). Figure 10 shows the relationship between output variability and government size (government revenues as a fraction of GDP). This figure suggests the presence of a negative link between government size and cyclical variability. Countries with a large public sector (*e.g.*, Sweden, Norway, France) typically experience less cyclical fluctuations than nations with small governments (like Japan, Spain, and Portugal). This is confirmed in a simple regression model in which output variability is regressed against government size (and an intercept term): the regression coefficient from an OLS procedure equals -0.066, with a standard error of 0.024. So a 10%-point increase in the government revenues/GDP ratio now leads to a 0.7%-point reduction of output fluctuations. The relationship between output variability and government size for the artificial economy is illustrated in Figure 11. In the experiment, we vary the unconditional mean of the capital tax rate, holding constant the unconditional mean of the labour tax rate and the standard deviation of innovations in technology and taxation. Consistent with the empirical data, the figure shows an inverse link between government size and macroeconomic instability: the regression coefficient from a similar OLS procedure equals -0.022, with a standard error of 0.0016. So the predicted interdependence between the size of the public sector and the intensity of business cycle fluctuations in the artificial economy is weaker than in reality. This may be due to cross-country differences in technology shocks and tax randomness, but a more detailed investigation is left for future research.

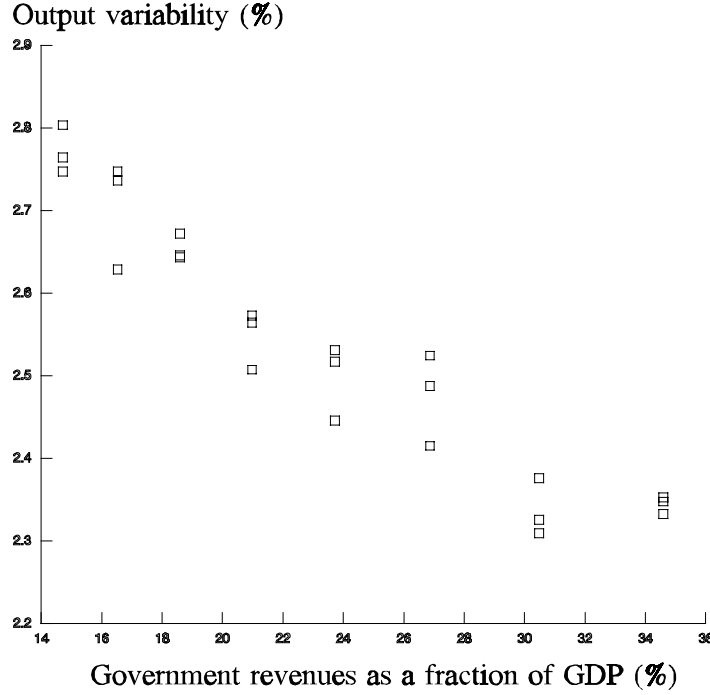


Note: The countries are USA, UK, Jap=Japan, Ger=Germany, Fra=France, Ita=Italy, Can=Canada, Tri=Austria, Bel=Belgium, Den=Denmark, Fin=Finland, Gre=Greece, Ice=Iceland, Ire=Ireland, Lux=Luxembourg, Net=The Netherlands, Nor=Norway, Por=Portugal, Spa=Spain, Swe=Sweden, Swi=Switzerland, Aus=Australia. Data on government revenues as a fraction of GDP are taken from Galí (1994). Output variability is measured as the standard deviation of real GDP per capita over the 1950-92 period, available from the Summers and Heston PWT 5.6 dataset. Figures on real GDP per capita are logged and detrended, setting the smoothing parameter at 400.

Figure 10: Government size and macroeconomic instability in the OECD.

10. Evaluation and conclusion

This paper investigated the interrelationship between long-term economic growth and cyclical variability. It is nowadays widely believed that public schooling may contribute favourably to long-term economic growth. But the income tax rates that are needed to finance such policy typically show an erratic time pattern. Such tax randomness could increase the intensity of the business cycle. Thus, government spending on education may spur economic growth, but the other side of the coin is that this is likely to increase the intensity of cyclical fluctuations. These issues were discussed in the context of a stochastic endogenous growth



Note: Government revenues as a fraction of GDP is calculated from $T/Y = \tau - \tau_k \delta/y$. HP-filtered output variability is derived from experiments with $\tau_k = 0; 10\%; 20\%; 30\%; 40\%; 50\%; 60\%; 70\%$, holding τ_N constant at 23%. The experiments (200 runs of 40 periods) are repeated three times.

Figure 11: Government size and macroeconomic instability in the artificial economy.

model with learning-by-doing as well as educational government spending. It turned out that tax randomness contributes modestly to the business cycle pattern. About 7% of output variability is due to stochastic tax rates in this model economy. We investigated the ability of our model to replicate some salient U.S. business cycle features. Whereas a version of our model that abstracts from tax stochasticity only performs fairly, introducing tax randomness improves the performance of the model. To bring aggregate labour market fluctuations more in line with the data, we proposed to introduce stochastic discounting. It has been shown that a theoretical economy may pass the Wald-test in that case. We finally discussed the model's implications for the interaction between economic growth and cyclical variability, and for the effect of government size on macroeconomic instability.

The analysis can be extended in various directions. The present paper started from a positive perspective by taking government behaviour as exogenously given. A first suggestion for future research is to incorporate the model within a normative framework. For instance,

the question of optimal taxation and optimal fiscal policy could be addressed. This certainly would be an ambitious project. The benevolent social planner should select optimal capital and labour tax rates, as well as an optimal spending program on lump-sum transfers and public services. A second question addresses the issue of an optimal Ramsey taxation scheme, for a given government spending program. A third interesting exercise would be an evaluation of the welfare effects when tax randomness is eliminated. Another interesting project would be to adopt more sophisticated econometric techniques (such as the Simulated Method of Moments) to test the model's ability to replicate important business cycle facts more thoroughly.

Appendix I: Balanced growth

Using the definitions for the transformed variables from section 3, we rewrite the equations that characterize the model in terms of transformed variables that are constant along a balanced growth path:

$$\gamma_t^K = R_t + (1 - \tau_{N,t})w_t N_t + \pi_t - c_t \quad (\text{A1})$$

$$\zeta = \frac{(1 - \tau_{N,t})w_t}{c_t} \quad (\text{A2})$$

$$1 = E_t \left[\beta_{t+1} \frac{c_t}{c_{t+1}} \frac{1}{\gamma_t^K} R_{t+1} \right] \quad (\text{A3})$$

$$y_t = A_t [z_{t-1} L_t]^{1-\alpha} \quad (\text{A4})$$

$$D_t = \alpha y_t \quad (\text{A5})$$

$$w_t = (1 - \alpha) \frac{y_t}{L_t} \quad (\text{A6})$$

$$R_t = 1 + (1 - \tau_{K,t})(D_t - \delta) \quad (\text{A7})$$

$$z_t \gamma_t^K = z_{t-1} (\chi + \psi L_t + \omega v_t) \quad (\text{A8})$$

$$N_t = L_t + v_t \quad (\text{A9})$$

$$v_t = g_t / w_t \quad (\text{A10})$$

$$\pi_t = \eta (\tau_r y_t - \tau_{K,t} \delta) \quad (\text{A11})$$

$$(1 - \tau_{N,t}) g_t = (1 - \eta) (\tau_r y_t - \tau_{K,t} \delta) \quad (\text{A12})$$

$$\tau_t = \alpha \tau_{K,t} + (1 - \alpha) \tau_{N,t} \quad (\text{A13})$$

As can easily be verified, A.1-A.13 directly follow from eqs. 2-6, 8-13, 15-16 and the definition of τ in the text. Since the transformed variables are constant along a balanced growth path, the set of equations characterizing equilibrium can easily be obtained by omitting time indices:

$$\gamma^* = R^* + (1 - \tau_N)w^*N^* + \pi^* - c^* \quad (\text{A1}')$$

$$\zeta = \frac{(1 - \tau_N)w^*}{c^*} \quad (\text{A2}')$$

$$1 = \beta \frac{R^*}{\gamma^*} \quad (\text{A3}')$$

$$y^* = A[z^*L^*]^{1-\alpha} \quad (\text{A4}')$$

$$D^* = \alpha y^* \quad (\text{A5}')$$

$$w^* = (1 - \alpha) \frac{y^*}{L^*} \quad (\text{A6}')$$

$$R^* = 1 + (1 - \tau_K)(D^* - \delta) \quad (\text{A7}')$$

$$\gamma^* = \chi + \psi L^* + \omega v^* \quad (\text{A8}')$$

$$N^* = L^* + v^* \quad (\text{A9}')$$

$$v^* = g^*/w^* \quad (\text{A10}')$$

$$\pi^* = \eta(\tau y^* - \tau_K \delta) \quad (\text{A11}')$$

$$(1 - \tau_N)g^* = (1 - \eta)(\tau y^* - \tau_K \delta) \quad (\text{A12}')$$

$$\tau = \alpha \tau_K + (1 - \alpha) \tau_N \quad (\text{A13}')$$

The proposed procedure to derive the balanced growth expression for the rate of economic growth is the following. Using A.3' we can eliminate R^* in the household's resource constraint A.1'. A11' is plugged into A1' to eliminate π^* . Using A.2' we can eliminate c^* in the resource constraint. Then we

divide the LHS and RHS of the resource constraint by $(1-\tau_N)w^*$. Making use of eq. A.6', we can eliminate w^* . From combination of A.3', A.5', and A.7' we can write y^* in terms of γ^* (and a subset of parameters). From A.8' we can write L^* in terms of γ^* and v^* (and a subset of parameters). Using A10', and A12' we can write v^* as a function of L^* and y^* . Substituting these expressions into the resource constraint and rearranging terms finally gives a complicated equation in γ^* ; this equation is solved numerically. Given γ^* , balanced growth expressions for the other variables in the model can easily be found.

Appendix II: Solving a loglinear stochastic version of the model

In order to study the extent to which tax randomness affects the business cycle, we solve a loglinearized version of our model with the method of undetermined coefficients (McCallum 1983, Campbell 1994, Uhlig 1995). Let $\tilde{y}_t \equiv \ln(y_t) - \ln(y^*)$ denote the log-deviation of $y_t = Y_t/K_{t-1}$ from its balanced growth value y^* , and define \tilde{c}_t , \tilde{N}_t , \tilde{L}_t , \tilde{v}_t , \tilde{w}_t , \tilde{R}_t , \tilde{D}_t , \tilde{z}_t , \tilde{A}_t , $\tilde{\tau}_{N,t}$, $\tilde{\tau}_{K,t}$, and $\tilde{\beta}_t$ in a similar way. Using Uhlig's (1995) methodology of loglinearization, it can be checked that the loglinearized model takes the form:

$$0 \approx \gamma^* \tilde{\gamma}_t^K - R^* \tilde{R}_t - (1 - \tau_N) w^* N^* \tilde{w}_t - (1 - \tau_N) w^* N^* \tilde{N}_t + \tau_N w^* N^* \tilde{\tau}_{N,t} - \pi^* \tilde{\pi}_t + c^* \tilde{c}_t \quad (\text{A1''})$$

$$0 \approx \tilde{w}_t - \tilde{c}_t \quad (\text{A2''})$$

$$0 \approx E_t[\tilde{\beta}_{t+1} + \tilde{c}_t - \tilde{c}_{t+1} - \tilde{\gamma}_t^K + \tilde{R}_{t+1}] \quad (\text{A3''})$$

$$0 = \tilde{y}_t - \tilde{A}_t - (1 - \alpha) \tilde{z}_{t-1} - (1 - \alpha) \tilde{L}_t \quad (\text{A4''})$$

$$0 \approx \tilde{D}_t - \tilde{y}_t \quad (\text{A5''})$$

$$0 \approx \tilde{w}_t - \tilde{y}_t + \tilde{L}_t \quad (\text{A6''})$$

$$0 \approx R^* \tilde{R}_t - (1 - \tau_K) D^* \tilde{D}_t + (D^* - \delta) \tau_K \tilde{\tau}_{K,t} \quad (\text{A7''})$$

$$0 \approx \gamma^* \tilde{z}_t - \gamma^* \tilde{z}_{t-1} + \gamma^* \tilde{\gamma}_t^K - \psi L^* \tilde{L}_t - \omega v^* \tilde{v}_t \quad (\text{A8''})$$

$$0 \approx N^* \tilde{N}_t - L^* \tilde{L}_t - v^* \tilde{v}_t \quad (\text{A9''})$$

$$0 \approx \tilde{v}_t - \tilde{g}_t + \tilde{w}_t \quad (\text{A10''})$$

$$0 \approx \pi^* \tilde{\pi}_t - \eta \tau y^* \tilde{\tau}_t - \eta \tau y^* \tilde{y}_t + \eta \delta \tau_K \tilde{\tau}_{K,t} \quad (\text{A11''})$$

$$0 \approx (1 - \tau_N) g^* \tilde{g}_t - \tau_N g^* \tilde{\tau}_{N,t} - (1 - \eta) \tau y^* \tilde{\tau}_t - (1 - \eta) \tau y^* \tilde{y}_t + (1 - \eta) \delta \tau_K \tilde{\tau}_{K,t} \quad (\text{A12''})$$

$$0 \approx \tau \tilde{\tau}_t - \alpha \tau_K \tilde{\tau}_{K,t} - (1 - \alpha) \tau_N \tilde{\tau}_{N,t} \quad (\text{A13''})$$

We finally write the linear system in matrix form, suitable to run the MATLAB software package developed by Uhlig (1995). The method in Uhlig (1995) yields the recursive equilibrium laws of motion in the form

$$\tilde{y}_t = \Lambda_{yz}\tilde{z}_{t-1} + \Lambda_{yS}\tilde{S}_t \quad (\text{A.14})$$

where Λ_{yz} is the partial elasticity of \tilde{y}_t with respect to \tilde{z}_{t-1} , and Λ_{yS} is a vector of partial elasticities of \tilde{y}_t with respect to \tilde{S}_t . \tilde{S}_t is a vector of the stochastic series for total factor productivity, capital taxation, and labour taxation: $\tilde{S}_t = [\tilde{A}_t \ \tilde{\tau}_{K,t} \ \tilde{\tau}_{N,t}]'$. Given balanced growth values y^* , c^* , N^* , L^* , v^* , w^* , R^* , D^* , and z^* , the method delivers the coefficients Λ_{yh} , Λ_{yA} , $\Lambda_{y\tau K}$, $\Lambda_{y\tau N}$, Λ_{ch} , Λ_{cA} , ..., *i.e.* the recursive equilibrium laws of motion

$$\tilde{z}_t = \Omega_P\tilde{z}_{t-1} + \Omega_Q\tilde{S}_t \quad (\text{A.15})$$

and

$$\begin{bmatrix} \tilde{y}_t \\ \tilde{c}_t \\ \tilde{N}_t \\ \tilde{L}_t \\ \tilde{v}_t \\ \tilde{w}_t \\ \tilde{R}_t \\ \tilde{D}_t \end{bmatrix} = \Omega_P\tilde{z}_{t-1} + \Omega_S\tilde{S}_t \quad (\text{A.16})$$

where Ω_P , ... , Ω_S are matrices containing the partial elasticities we are looking for.

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